6.2: Cardinality

Definition 1. If A is a finite set, then its cardinality is the number of elements in A. We denote the cardinality by $n(A)$ or $|A|$.

Example 1.

(a) Let $S = \{a, b, c\}$. Then $|S| = 3$.

(b) Let A be the set of outcomes when rolling one die. Then $n(A) = 6$.

Cardinality of a Union

Question 1. How can we calculate $n(A \cup B)$ if we know $n(A)$ and $n(B)$? Give an initial guess and then try the next example.

Example 2. Let $A = \{a, b, c\}$ and $B = \{b, c, d\}$. Find $n(A)$, $n(B)$ and $n(A \cup B)$. Was your guess correct? If not, what needs to be changed?

Cardinality of a Complement

Question 2. Let S be a finite universal set containing A. Give an initial guess for $n(A')$.

Example 3. Let S be the possible outcomes of rolling a single die. Let A be the subset of S which contains all rolls strictly greater than 2. Find $n(A)$, $n(S)$ and $n(A')$. Was your guess correct?

Cardinality of a Cartesian Product

If A and B are sets, then $n(A \times B) = n(A) \cdot n(B)$.

Example 1. Let S be the sample space when rolling a single die. Let A be the event that an odd number is rolled.

(a) What is the cardinality of $S \times S$ and what does it represent?

(b) What is the cardinality of $A \times A$ and what does it represent?

(c) What is the cardinality of $S \times A$ and what does it represent?

DeMorgan's Laws Let $A_1, A_2, \ldots, A_n$ be sets.

1. $(A_1 \cup A_2 \cup \cdots \cup A_n)' = A_1' \cap A_2' \cap \cdots \cap A_n'$
2. $(A_1 \cap A_2 \cap \cdots \cap A_n)' = A_1' \cup A_2' \cup \cdots \cup A_n'$